

# 17 | Tychonoff Theorem

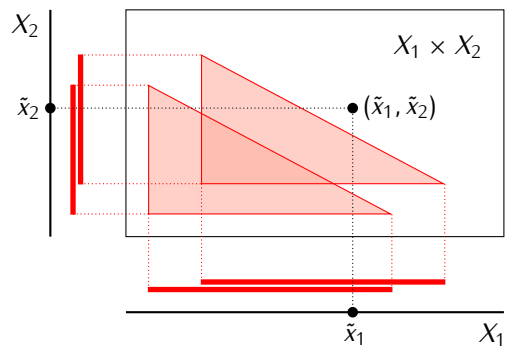
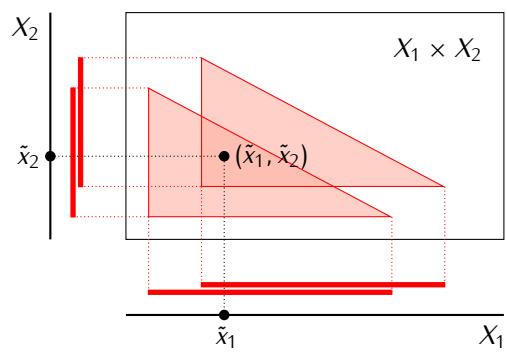
**17.1 Tychonoff Theorem.** *If  $\{X_s\}_{s \in S}$  is a family of topological spaces and  $X_s$  is compact for each  $s \in S$  then the product space  $\prod_{s \in S} X_s$  is compact.*

**17.2 Definition.** Let  $\mathcal{A}$  be a family of subsets of a space  $X$ . The family  $\mathcal{A}$  is *centered* if for any finite number of sets  $A_1, \dots, A_n \in \mathcal{A}$  we have  $A_1 \cap \dots \cap A_n \neq \emptyset$

**17.5 Lemma.** *Let  $X$  be a topological space. The following conditions are equivalent:*

1) *The space  $X$  is compact.*

2) *For any centered family  $\mathcal{A}$  of closed subsets of  $X$  we have  $\bigcap_{A \in \mathcal{A}} A \neq \emptyset$ .*



**17.16 Proposition.** *If  $X_i$  is a Hausdorff space for each  $i \in I$  then the product space  $\prod_{i \in I} X_i$  is also Hausdorff.*

*Proof.* Exercise. □

**17.17 Corollary.** *If  $X_i$  is a compact Hausdorff space for each  $i \in I$  then the product space  $\prod_{i \in I} X_i$  is also compact Hausdorff.*